

Block IV Subcarrier Demodulator Assembly Acquisition Problem

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The Block IV Subcarrier Demodulator Assembly (SDA) has been designed with four loop bandwidths. Two of these bandwidths are designed with unity damping, while the other two are achieved by increasing the loop gain. Normal tracking can be done in any one of the four bandwidths, or the "high-gain" bandwidths of the set may be used as an acquisition bandwidth. The transition from acquisition to tracking mode should be accomplished by providing a slow reduction in gain in order to limit the peak phase error during the transition time. Excessive phase errors can lead to loss of lock or a greatly diminished quality of data. Experimental evidence and preliminary analysis have shown that the original phase error transient due to acquisition should be allowed to die out before bandwidth reduction is initiated in order to minimize the peak phase error. This article documents the experiments and analysis that led to the bandwidth reduction procedure used in the Block IV SDA so that acquisition is complete 80 seconds after phase lock for the 3.9- to 0.5-Hz configuration and 1300 seconds after phase lock for the 0.23- to 0.03-Hz configuration.

I. Introduction

The Block IV Subcarrier Demodulator Assembly (SDA) has been designed with four loop bandwidths: 0.03, 0.23, 0.5, and 3.9 Hz. Two of these bandwidths (0.03 and 0.5 Hz) have been designed with critical damping and will be called narrow-bandwidth, normal-gain and wide-bandwidth, normal-gain, respectively. These two bandwidths can be increased to 0.23 and 3.9 Hz by increasing

the gain by ten to one, and are then called narrow-bandwidth, high-gain and wide-bandwidth, high-gain.

Two fundamental acquisition techniques are possible: (1) change the loop time constants and the loop gain (i.e., keep the damping factor constant) and (2) change only the loop gain (increase the damping factor when the loop gain is raised). The first approach was not used since it is

more difficult to implement. Furthermore, a smooth bandwidth transition is not practical to implement when both time constant and gain are changed in such a manner as to keep the damping constant. The second approach of gain reduction to effect a bandwidth reduction can be accomplished either by an instantaneous gain switch or a gradual change in the gain.

Figure 1 illustrates a series of tests (all testing discussed in this article will be limited to the more difficult narrow-bandwidth case) in which the phase transient was measured after T_1 seconds had elapsed between the last cycle slip and when the gain was switched instantaneously from the high value to the normal value. In this case, the results of this test indicate that if T_1 is made large enough (approximately 1000 seconds), the transient can be held to an acceptably low level, since in the range of anticipated frequency offsets, the effect due to frequency offset is negligible. This test was repeated in the presence of design point noise ($S/N_0 = +8$ dB), and the peak phase error increased to an unacceptable level.

To gain an insight into the problem, a control loop was developed such that the loop gain was held high until the error was reduced to some preset value (ϕ_ϵ), and then the loop gain was allowed to reduce in such a fashion as to keep the phase error constant at ϕ_ϵ . This experiment (see Fig. 2) illustrates that not only must we wait T_1 seconds for the acquisition transient to die out, but that we require T_2 seconds to reduce the gain. Further, it establishes an experimental value for T_2 that will be useful in confirming the analysis that follows.

The preferred acquisition technique is to acquire in the narrow-bandwidth, high-gain mode and allow the acquisition transient T_1 seconds to die out and then smoothly reduce the bandwidth to the narrow-bandwidth, normal-gain mode in T_2 seconds.

Two more experiments were run to confirm the relationship between T_1 and T_2 and the peak phase error. Figure 3 shows T_2 held constant and illustrates that, even with a "soft switch," the initial transient must be allowed to die out. The experiment shown in Fig. 4 was run holding T_1 constant and illustrates the importance of properly controlling the time T_2 allowed for the gain reduction.

II. Analysis

The two aspects of the problem that are analyzed here are the transient settling time T_1 and the time to reduce the gain to 10% of the initial value, which is denoted by

T_2 . Both calculations are based on the assumption that there is no noise present in the loop. A study is in progress to consider the case in which there is noise. Based on the experimental evidence of Figs. 1–4, it is anticipated that, even at threshold, the two times T_1 and T_2 will not change appreciably from those predicted in this analysis.

A. Transient Response

In order to make the problem amenable to solution, we assume that the phase error is always small enough so that the linear model is applicable. Figure 5 shows the base-band model of a linear phase-locked loop.

If the loop filter is given by

$$F(s) = \frac{(1 + \tau_2 s)^2}{(1 + \tau_1 s)^2} \quad (1)$$

then the differential equation describing the loop phase error $\phi(t)$ is given by

$$\tau_1^2 \ddot{\phi}(t) + (2\tau_1 + G\tau_2^2) \dot{\phi}(t) + (1 + 2G\tau_2) \phi(t) + G\phi(t) = \ddot{\theta}_i + 2\tau_1 \dot{\theta}_i + \tau_1^2 \ddot{\theta}_i \quad (2)$$

where $G = K_v K$ is the open-loop gain of the loop and contains the limiter suppression gain as well. The case of interest occurs when the input has a doppler shift Ω_0 , so that

$$\frac{d\theta}{dt} = \Omega_0 \quad (3)$$

In order to specify a solution, the three initial conditions must be specified:

$$\begin{aligned} \phi(0) &= \phi_0 \\ \dot{\phi}(0) &= \dot{\phi}_0 \\ \ddot{\phi}(0) &= \ddot{\phi}_0 \end{aligned} \quad (4)$$

Following frequency acquisition, we can arbitrarily define $t = 0$ to be the time at which the initial conditions are specified. Taking Laplace transforms of Eq. (2), we find that the Laplace transform of the phase error $\Phi(s)$ is given by

$$\begin{aligned} \Phi(s) &= \frac{\Omega_0/s + \tau_1^2 \ddot{\phi}_0 + [\tau_1^2 s + (2\tau_1 + G\tau_2^2)] \dot{\phi}_0}{\tau_1^2 s^3 + (2\tau_1 + G\tau_2^2) s^2 + (1 + 2G\tau_2) s + G} \\ &+ \frac{[\tau_1^2 s^2 + (2\tau_1 + G\tau_2^2) s + (1 + 2G\tau_2)] \phi_0}{\tau_1^2 s^3 + (2\tau_1 + G\tau_2^2) s^2 + (1 + 2G\tau_2) s + G} \end{aligned} \quad (5)$$

From the final value theorem, we can show that the steady-state phase error is given by

$$\phi_{ss} = \lim_{t \rightarrow \infty} \phi(t) = \lim_{s \rightarrow 0} S\Phi(s) = \frac{\Omega_0}{G} \quad (6)$$

In order to obtain the inverse Laplace transform of $\Phi(s)$, we must factor the denominator of the $\Phi(s)$ polynomial in Eq. (5). Using the parameters of the narrow-band, high-gain mode, $G = 568$, $\tau_1 = 5,250$, and $\tau_2 = 148$, and the use of a root finding program the phase error was found to be given by

$$\phi(t) = 0.0176\Omega_0 + Ae^{-0.00603t} + Be^{-0.44075t} + Ce^{-0.00775t} \quad (7)$$

where A , B , and C depend on the loop parameters as well as the initial conditions ϕ_0 , $\dot{\phi}_0$, and $\ddot{\phi}_0$, which are unknown. It has been found experimentally that it is necessary to wait until the phase error and its derivatives are very small before the gain is reduced to narrow the loop bandwidth. It is clear that if the first exponential term is reduced to, say, 2% of its final value by waiting until some time T_1 , then at that time, $\phi(t)$ is less than or equal to 2% of its initial value. If we arbitrarily select 2% (3.9 time constants) as the required value, then we must wait

$$T_{1(\text{narrow})} = 648 \text{ seconds} \quad (8)$$

before the gain is reduced. This value of T_1 has been verified experimentally (see Fig. 3).

In the wide-band, high-gain mode, using the parameters $\tau_1 = 5,250$, $\tau_2 = 8.91$, and $G = 2.63 \times 10^6$, we find that

$$\phi(t) = 3.85 \times 10^{-7} \Omega_0 + A'e^{-0.10063t} + B'e^{-0.12909t} + C'e^{-0.73454t} \quad (9)$$

where, as before, A' , B' , and C' depend on the above loop parameters as well as ϕ_0 , $\dot{\phi}_0$, and $\ddot{\phi}_0$. We find that the phase error is no larger than 2% of its maximum value just after frequency acquisition in the wide-band acquisition mode when $t = T_1$, where

$$T_{1(\text{wide})} = 40 \text{ seconds}$$

B. Required Gain Change to Maintain Constant Phase Error

After the phase error is reduced to a sufficiently small value following acquisition, the gain is reduced to bring

the loop into the narrow-bandwidth, normal-gain mode. Generally speaking, a quick reduction of the gain causes the phase error to increase dramatically, with the attendant possibility of losing lock (see Figs. 1 and 4). A reasonable approach to the gain reduction problem is to reduce the gain at a rate such that the phase error does not exceed some maximum value. Under a maximum phase error constraint, the optimal procedure for minimizing the time to reduce the gain to 10% of its initial value is to hold the phase error constant at the maximum value by reducing the gain accordingly. We now derive the loop equation in the case in which the loop gain is a function of time, so that we can determine the optimal gain contour and T_2 . In Fig. 6, each low-pass filter has the transfer function

$$H(s) = \frac{1 + \tau_2 s}{1 + \tau_1 s} \quad (11)$$

To simplify the calculations, we let (using $\tau_2 \ll \tau_1$)

$$H(s) \cong \alpha + \frac{1}{\tau_1 s}, \quad \alpha = \frac{\tau_2}{\tau_1} \quad (12)$$

We assume that $G(t)$ is an arbitrary function of time. Then, assuming that the initial voltages v_0 and v_1 are stored on the capacitors, we have (for convenience letting $t = 0$ represent the time T_1 s after acquisition)

$$\begin{aligned} \theta - \phi = & \int_0^t \left\{ v_1 + \alpha \left[v_0 + \alpha AK\phi + \frac{1}{\tau_1} \int_0^{t'} AK\phi(t'') dt'' \right] \right. \\ & \left. + \frac{1}{\tau_1} \int_0^{t'} \left[v_0 + \alpha AK\phi + \frac{1}{\tau_1} AK\phi \right] dt'' \right\} dt' \end{aligned} \quad (13)$$

If we normalize time by

$$\tau = \frac{t}{\tau_2} \quad (14)$$

and define a new gain parameter $q(t)$ (the gain contour) by

$$q(t) = \frac{G(t) \tau_2^3}{\tau_1^2} \quad (15)$$

then, by suitable differentiation, we can show that Eq. (13) becomes

$$\begin{aligned} \phi(\tau) \ddot{q}(\tau) + (2\dot{\phi}(\tau) + 2\phi(\tau)) \dot{q}(\tau) \\ + (\ddot{\phi}(\tau) + 2\dot{\phi}(\tau) + \phi(\tau)) q(\tau) = \ddot{\theta}(\tau) - \ddot{\phi}(\tau) \end{aligned} \quad (16)$$

In the case where the phase error and the input phase are constant, the differential equation becomes

$$\ddot{q}(\tau) + 2q(\tau) + \dot{q}(\tau) = 0 \quad (17)$$

The solution is given by

$$q(\tau) = (A + B\tau)e^{-\tau}, \quad \tau \geq 0 \quad (18)$$

In order to evaluate the unknown constants A and B , we note that at $\tau = 0$ (the time when the phase error is first held constant), the charge on the capacitors is constant. By differentiating Eq. (13) once, normalizing using Eq. (14), and letting $\tau = 0$, we get

$$\dot{\theta}(0) - \dot{\phi}(0) - q(0)\phi(0) = (v_1 + \alpha v_0)\tau_2 \quad (19)$$

Since the phase error $\phi(t)$ is continuous near zero and v_0 and v_1 are constant at $\tau = 0$, we see that the right-hand side of Eq. (19) is constant. Letting the time 0^- denote the time just before the phase error is held constant and 0^+ the time just after, we obtain from Eq. (19) that

$$q(0^+) = q(0^-) + \frac{\dot{\phi}(0^-)}{\phi(0^-)} \quad (20)$$

By differentiating Eq. (13) twice and using Eq. (14), we have, at $\tau = 0$,

$$\ddot{\theta}(0) - \ddot{\phi}(0) - \dot{q}(0)\phi(0) - q(0)\dot{\phi}(0) - 2q(0)\phi(0) = v_0\alpha\tau_2 \quad (21)$$

Since the right-hand side is constant at $\tau = 0$, we can show that

$$\dot{q}(0^+) = \frac{\ddot{\phi}(0^-)}{\phi(0^-)} + \frac{\dot{\phi}(0^-)}{\phi(0^-)} [q(0^-) - 2] \quad (22)$$

Hence, in order that the phase error be held constant, we require (converting derivatives on τ to derivatives on

time) that the optimal gain contour using Eqs. (20) and (22) in Eq. (18) is given by

$$q(t) = \left\{ q(0^-) + \tau_2 \frac{\dot{\phi}(0^-)}{\phi(0^-)} + \left[q(0^-) + \tau_2^2 \frac{\ddot{\phi}(0^-)}{\phi(0^-)} + \tau_2 \frac{\dot{\phi}(0^-)}{\phi(0^-)} (q(0^-) - 1) \right] \frac{t}{\tau_2} \right\} e^{-t/\tau_2} \quad (23)$$

which is valid for $t > 0^+$.

The initial conditions are in general unknown. However, under the assumption that we wait until the phase error has decayed to about 2% of its maximum value obtained after frequency acquisition, it can be shown that both

$$\tau_2 \frac{\ddot{\phi}(0^-)}{\phi(0^-)}$$

and

$$\tau_2 \frac{\dot{\phi}(0^-)}{\phi(0^-)}$$

are close to unity and $q(0^-)$ is 67.5, which is obtained by using Eq. (15) and $G = 568$, $\tau_1 = 5250$, and $\tau_2 = 148$, so that in the narrow-band, high-gain mode we have approximately

$$q(t) = \left(68 + \frac{134t}{\tau_2} \right) e^{-t/\tau_2} \quad (24)$$

Since the bandwidth depends on $q(t)$ by

$$w_l = 0.115w_{l_0} q \left[\frac{q + 1.5}{q - 0.5} \right] \quad (25)$$

we see that a 10 to 1 reduction in $q(t)$ provides a 7.8 to 1 reduction in bandwidth. Using Eq. (24) we find that for $q(t)$ to drop to 6.75 (10% of 67.5) requires 680 seconds in the narrow-band mode and 41 seconds in the wide-band mode. The actual gain contour used for bandwidth reduction is an approximation to Eq. (24).

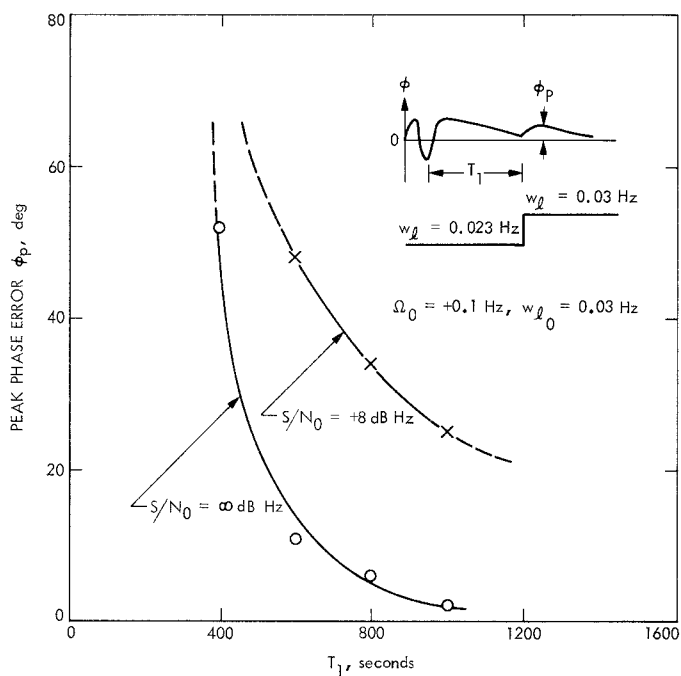


Fig. 1. Peak phase error vs. settling time T_1 for a step change in bandwidth

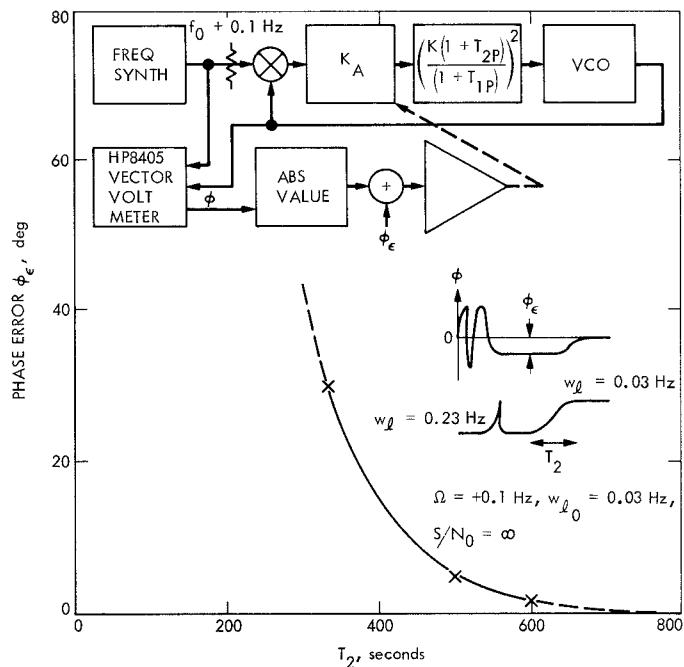


Fig. 2. Controlled phase error ϕ_e vs. time T_2 to reduce gain to 10% of initial value

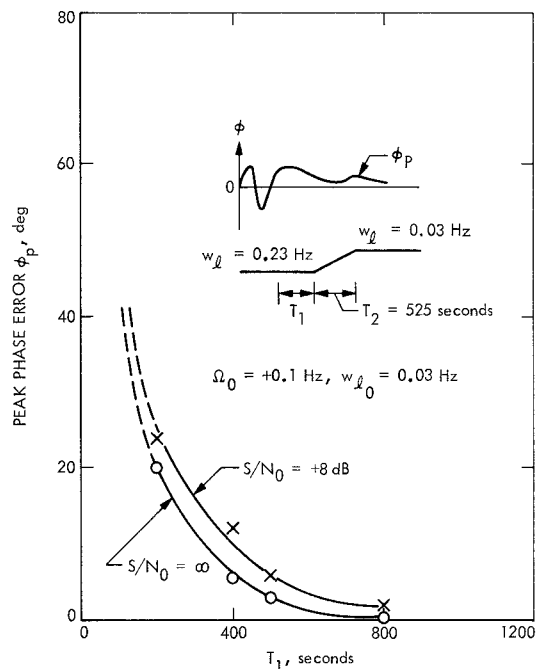


Fig. 3. Peak phase error vs. T_1 for a gradual change ($T_2 = 525$ s) in bandwidth

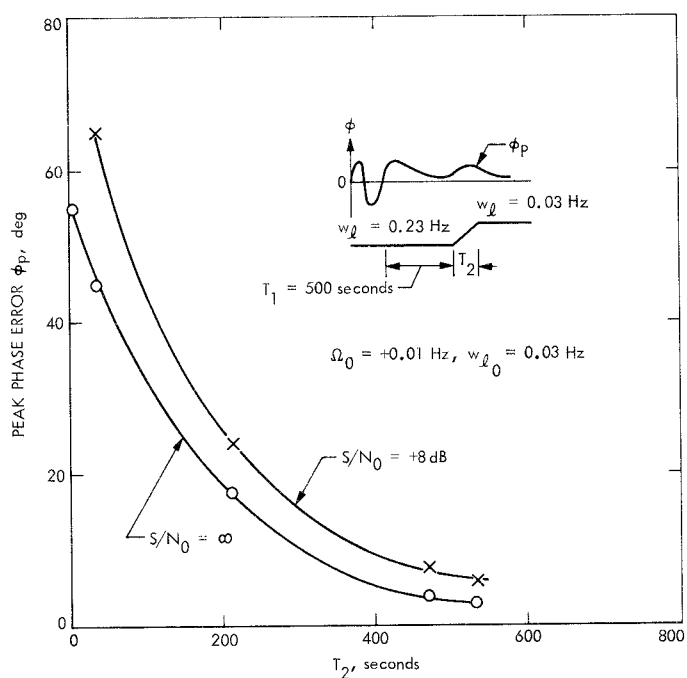


Fig. 4. Peak phase error vs. T_2 after phase transient is allowed to die out (T_1) for 500 s

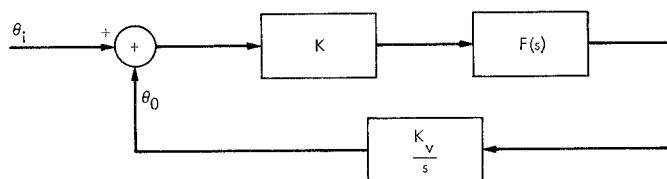


Fig. 5. Linear model of phase-locked loop

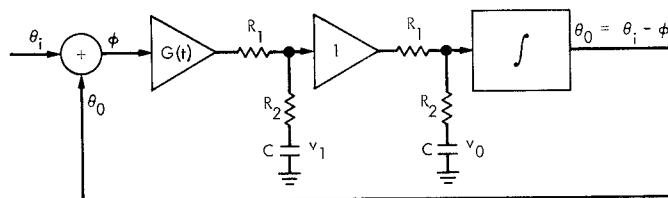


Fig. 6. Model of third-order loop showing initial capacitor voltages